习题 1 证明：$f_k \rightrightarrows f$ on $A \iff \exists \{k_l\}_{l=1}^{\infty}$, s.t. $A = \bigcap\limits_{l=1}^{\infty} \bigcap\limits_{k=k_l}^{\infty} \left\{|f_k - f| < \frac{1}{l}\right\}$

提示：回忆一致收敛的定义，我们有

$$\begin{aligned} f_k \rightrightarrows f \text{ on } A &\iff \forall \varepsilon > 0, \exists N, \text{s.t. } \sup_{x \in A} |f_k(x) - f(x)| < \varepsilon, \forall k \geq N \\ &\iff \forall l \in \mathbb{N}^*, \exists k_l, \text{s.t. } \sup_{x \in A} |f_k(x) - f(x)| < \frac{1}{l}, \forall k \geq k_l \end{aligned}$$

习题 2（Stein,Ch1,T22）Let $\chi_{[0,1]}$ be the characteristic function of $[0,1]$. Show that there is no everywhere continuous function f on $\mathbb{R}$ such that

$$f(x) = \chi_{[0,1]}(x)$$

almost everywhere.

习题 3（Stein,Ch1,T23）Suppose $f(x,y)$ is a function on $\mathbb{R}^2$ that is separately continuous: for each fixed variable, f is continuous in the other variable. Prove that f is measurable on $\mathbb{R}^2$.

Hint: Approximate f in the variable x by piecewise-linear functions f_n so that $f_n \to f$ pointwise.