习题 1 设 $f: E \to [-\infty, +\infty]$，定义原像集 $f^{-1}(A) = \{x \in E : f(x) \in A\}$，证明：以下等价 TFAE(The Followings Are Equivalence)

(1) f 可测

(2) $\forall G \overset{\text{open}}{\subset} \mathbb{R}, f^{-1}(G) \in \mathscr{L}$

(3) $\forall F \overset{\text{closed}}{\subset} \mathbb{R}, f^{-1}(F) \in \mathscr{L}$

(4) $\forall B \in \mathcal{B}_{\mathbb{R}}, f^{-1}(B) \in \mathscr{L}$

习题 2 证明：若 $g: E \to [-\infty, +\infty]$ 可测，则 $\frac{1}{g}$ 可测（这里我们假设 $g(x) \neq 0, \forall x \in E$）

习题 3 (Stein,T35) (1).Give an example of a measurable function f and a continuous function Φ so that $f \circ \Phi$ is non-measurable.

[Hint: Let $\Phi : \mathcal{C}_1 \to \mathcal{C}_2$ as in Exercise 34, with $m(\mathcal{C}_1) > 0$ and $m(\mathcal{C}_2) = 0$. Let $N \subset \mathcal{C}_1$ be non-measurable, and take $f = \chi_{\Phi_N}$.]

(2).Use the construction in the hint to show that there exists a Lebesgue measurable set that is not a Borel set.

习题 4 举例说明：$|f|$ 可测 $\not\Longrightarrow f$ 可测

习题 5 设 $f_k: E \to [-\infty, +\infty], \{f_k\}_{k=1}^{\infty}$ 可测，且 $f_k \to f$ a.e $x \in E$，求证：f 可测